A MODEL FOR PREDICTING THE GROWTH OF LISTERIA MONOCYTOGENES IN PACKAGED WHOLE MILK

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Accepted for Publication May 9, 2001

ABSTRACT

Experiments were conducted to determine growth characteristics of Listeria monocytogenes in sterilized whole milk at nine temperatures in the range of 277.15 to 308.15K (4 to 35°C). Based on these data, the parameter values of the Baranyi dynamic growth model were statistically determined. Finite element software, ANSYS, was used to determine temperature distributions in milk cartons subject to a time-varying ambient temperature profile. The space-time-temperature data were input to the Baranyi dynamic growth model, to predict the microbial population density distribution and the average population density in the milk carton. The Baranyi dynamic growth model and the finite element model were integrated and validated using experimental results from inoculated sterilized whole milk in half-gallon laminated paper cartons. In all experiments, the milk cartons were subjected to the same temperature profile as the Baranyi dynamic growth model. Experimental microbial counts were within predicted upper and lower bounds obtained using the integrated Baranyi dynamic growth and finite element models. In addition, the growth curve at the mean value of initial physiological state parameter for L. monocytogenes underpredicted the microbial growth (standard error = 0.54 log (cfu/mL) and maximum relative difference = 15.49%).

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INTRODUCTION

_listeria monocytogenes_

_Listeria monocytogenes_ is a psychrotrophic microorganism, which has been and continues to be of concern to food microbiologists. Several outbreaks of listeriosis (disease associated with this organism) in the past 15 years have been traced back to consumption of contaminated food products (Schlech _et al._ 1983; Fleming _et al._ 1985; Linnan _et al._ 1988; Radhakrishnan and Puri 1999), which include coleslaw, meat products, pasteurized milk, and milk products like cheese.

Postpasteurization microbial safety of milk and milk products depend, to a large extent, on refrigeration. Refrigerated storage, however, is not adequate for inhibiting growth of _L. monocytogenes_ in milk as it can multiply at refrigeration temperatures (i.e., in the range of 273.15K (0°C) to 281.15K (8°C)) (Donnelly and Briggs 1986; Rosenow and Marth 1987; Walker and Stringer 1987; Juntilla _et al._ 1988). Extended periods of refrigerated storage (up to 14 days are allowed for pasteurized milk) and inadequate compliance with temperature regulations during transportation and storage at retail stores (Barnard 1974 and 1992) may allow a few _L. monocytogenes_ cells that survived pasteurization or were added by postpasteurization contamination to attain populations high enough (10^2 cfu/mL or above) to pose serious health risks. Consequently, study of microbiological safety of packaged fluid milk in the post-pasteurization stage has great health significance.

Predictive Microbiology and the Baranyi Dynamic Growth Model

Traditional microbiological methods for determining food quality and safety are time consuming. New technology has made possible rapid indirect methods that often require use of sophisticated and expensive equipment (McMeekin _et al._ 1993). On the other hand, predictive models provide fast and relatively inexpensive ways to get reliable ‘first estimates’ of microbial growth and survival for prescribed conditions. That technique is becoming increasingly important as a powerful tool in food microbiology and safety. Predictive modeling can be used for: (1) describing behaviors of microorganisms at different physical and chemical conditions, and (2) process design and optimization of production and distribution chains, based on product microbial quality assurance and shelf-life. Predictive modeling involves the use of mathematical equations to describe microbial behavior, i.e., changes in population with time. Generally, these mathematical models include environmental conditions like temperature, pH, water activity and presence of inhibitory chemicals as parameters.
Several mathematical models predict lag times and growth rates of microorganisms at constant temperatures, but only a few researchers have dealt with predicting growth in situations where temperature is a dynamic variable. Blankenship et al. (1988) developed a dynamic model for predicting growth of Clostridium perfringens in cooked chili during chilling. The study utilized a spatially lumped time-explicit approach. Van Impe et al. (1995) used a time-implicit model for predicting growth of Brochothrix thermosphacta and Lactobacillus planatarum under time-varying temperature profiles. Baranyi et al. (1995) developed a mechanistic dynamic model as a set of differential equations, which included a parameter representing the physiological state of the cell culture ($a_0$). This model could predict growth successfully of Brochothrix thermosphacta in the range 278.15-295.15K (5-25°C) when temperature changed gradually and with frequent sudden changes.

The development of a dynamic model for growth of L. monocytogenes in fluid milk was described in detail by Alavi et al. (1999). Alavi et al. validated the Baranyi dynamic growth model (BDGM) for rapidly fluctuating temperature profiles. However, temperatures inside a milk carton or any other food product subject to dynamic temperature vary spatially as well as temporally. Spatial variation of temperature also affects the spatial distribution of microbial cells inside a product. From the above reasoning, the BDGM alone cannot predict growth of L. monocytogenes in a milk carton, it is necessary also to have a model which can predict both the temperature and microbial population distributions inside a milk carton.

Use of Finite Element Method in Heat Transfer Problems

The finite element method (FEM) is a common numerical technique applied to heat transfer problems in food and agricultural engineering (Puri and Anantheswaran 1993). Gustafson et al. (1977, 1979) utilized FEM to study the heating and cooling of corn kernels and its influence on stresses in nonhomogeneous regions (endosperm and germ) of the kernel. The FEM formulation included solution of the two-dimensional, time-dependent heat-conduction equation. The diffusion equation (steady and unsteady) was used by Potluri (1985) to develop a two-dimensional steady state finite element model to predict the cooling of loin carcasses and their surface-heat-transfer coefficients. Measured and FEM-predicted values of temperature were within 1°C, with $R^2$ greater than 0.92.

Misra and Young (1979) modeled the cooling of an apple in spherical coordinates by using a one-dimensional, time-dependent finite-element formulation. Comparison of FEM-predicted temperature values with a constant-property analytical solution indicated statistical similarity. Bazan et al. (1989) and Pan and Bhowmik (1991) used FEM to predict temperature distribution
during room cooling of a confined bin of spherical fruit, and of individual green tomatoes, respectively. Experimental results, in both cases, agreed well with predicted values. Carroll et al. (1996) used FEM to calculate the cooling rate of irregularly shaped fruit products — Roman apples and Bartlett pears.

Zhou (1993) and Zhou et al. (1995) developed a three-dimensional finite element model for microwave heating of food products; it may be used to predict temperature and moisture distributions, nutrient retention, and microbial destruction in food products. A modified version of that model was used by Vilayannur et al. (1998a, b) to determine the effect of size on uniformity of temperature and moisture distributions during microwave heating of food materials.

Developments of FEM-based software packages have made implementation of the method easier for various conditions encountered in food processing. FEM software packages that have been used for solving heat transfer problems in food and agricultural engineering include DOT (Determination of Temperature) (Jiang et al. 1987), TWODEPEP (Lin et al. 1995), FIDAP (Kumar et al. 1990) and ANSYS (Stringer et al. 1989) among others.

In this study, heat transfer, i.e., temperature distribution, in milk cartons was modeled using commercial finite element software ANSYS (Swanson Analytical Systems, Inc., Houston, PA). ANSYS was selected because of its availability and a user-friendly, menu-based interface.

**OBJECTIVES**

The overall goal of this study was to determine the feasibility of using the FEM in conjunction with the Baranyi dynamic growth model (BDGM) to predict the growth of *L. monocytogenes* in milk cartons in a dynamic temperature environment. The specific objectives were to:

1. Use finite element software ANSYS to predict the space-time temperature distribution $T(x,y,z,t)$ in packaged, sterilized fluid (whole) milk subject to a dynamic temperature regime as the ambient condition;
2. Predict from the temperature distribution obtained in Objective 1 as input to the BDGM the spatial distribution and average values of the population density for *L. monocytogenes* as functions of time. The combination of finite element software and the dynamic growth model is referred to as the integrated dynamic growth-finite element model (IDG-FEM);
3. Validate the IDG-FEM using experimental data of dynamic temperature and population density of *L. monocytogenes* in sterilized whole milk in a standard half-gallon laminated paper carton subject to the pseudo-random time-varying temperature profile used in Objective 1.
THEORY AND METHODOLOGY

Dynamic Microbial Growth Model

The current study utilized the approach developed by Baranyi et al. because of its advantages over other models: (1) sound theoretical basis, and (2) ability to handle rapid time-varying temperature change (Alavi et al. 1999). The theory of the BDGM was described by Baranyi et al. (1995). Alavi et al. (1999) reported the successful applications and validation of BDGM for growth of L. monocytogenes in fluid, whole milk. The BDGM consists of the two differential Eq. (1) and (2) below.

\[ \frac{dx}{dt} = \mu_{\text{max}} \left( \frac{q}{q+1} \right) \left( 1 - \frac{x}{x_{\text{max}}} \right)^m \]

\[ \frac{dq}{dt} = vq \]

where, \( x(t) \) is the cell concentration, in cfu/mL, \( q(t) \) represents the concentration of a substance critical to cell growth (such as RNA), \( v \) is the rate of growth of that critical substance, and \( m \) is the curvature parameter to characterize the transition from the exponential phase to the stationary phase. Additional discussion on curvature parameter, \( m \), is given in Baranyi et al. (1995) and Alavi et al. (1999). The term \( \frac{q}{q+1} \) in Eq. (1) is known as the physiological state parameter \( \alpha \) (0 ≤ \( \alpha \) ≤ 1), with initial value \( \alpha_0 \) describing the physiological state of the bacterial cells at the time of inoculation. Additional discussion on \( \alpha_0 \) may be found in Baranyi et al. (1995) and Alavi et al. (1999). Essentially, the variable \( q \) is expressed in terms of the physiological state parameter \( \alpha \). The maximum specific growth rate \( \mu_{\text{max}} \) of the cells (h⁻¹) and its dependence on temperature of the growth medium were described by the Zweitering square root Eq. (3), the parameters of which were found by curve fitting isothermal growth data by Alavi et al. (1999).

\[ \mu_{\text{max}} = [0.0185 (T-271.42)]^2 \{1 - \exp[T - 317.96]\} \]

The following assumptions were made to simplify the BDGM

(1) As discussed and recommended by Alavi et al. (1999) \( m \) was set to 1,
(2) The growth rate $\mu_{\text{max}}$ is not greater than that of the critical substance ($\nu$), which limits the growth, therefore, as a limiting value, $\mu_{\text{max}} = \nu$.

The BDGM was successfully validated by Alavi et al. (1999) using experimental data obtained for growth of *L. monocytogenes* in fluid whole milk contained in 20 mL test tubes and subjected to a dynamic temperature condition. The calculated values had a maximum relative error of 10.42% and a root mean square error of 0.28 log (cfu/mL) (Alavi et al. 1999). Equation (1)-(3) were used in this study to predict the growth of *L. monocytogenes* in whole milk.

**Governing Equation, Boundary and Initial Conditions for Temperature Distributions**

Temperature changes in the milk carton during storage were mathematically modeled as a conductive process and that convection is minimal. This assumption is especially valid for conditions where the temperature of the top surface of the enclosed fluid is higher than the bottom surface. In that case, the lower-density fluid is above the higher-density fluid, and no convection currents will occur (White 1988). Temperature of top surface > bottom surface occurred in this study when milk cartons were immersed in water baths. The upper surface of the milk in the carton was exposed to room temperature (299.15K or 26°C) while the lower surface was the lower temperature (285.45K or 12.3°C) water bath.

The governing equation for heat transfer in a milk carton is simply the conduction equation:

$$\rho \ C_p \ (\partial T/\partial t) = \nabla \cdot (k_T \ \nabla T)$$  \hspace{1cm} (4)

where

- $\rho$ is the density of whole milk = 1032 kg/m$^3$
- $C_p$ is the specific heat of whole milk = 3831 J/kg-K
- $k_T$ is the thermal conductivity of whole milk = 0.575 W/m-K
- $T$ is the temperature distribution within the milk carton (K), and
- $t$ is the time (s)

The boundary condition for Eq. (4) is given by heat flux Eq. (5) (for heat transfer due to convection).

$$k_T \ \nabla T \cdot n = U \ (T_s - T_{\text{bulk}})$$  \hspace{1cm} (5)
where

\[ U \] is the overall convective heat transfer coefficient (W/m²-K) and is calculated for the circulating water and the air in the head space of the carton separately as shown below.

\[ n \] is the unit outward normal vector

\[ T_s \] is the surface, i.e., boundary surface temperature (K)

\[ T_{\text{bulk}} \] is the temperature of the surrounding fluid (K)

The initial \((t=0)\) temperature of the milk sample was assumed to be uniform, \(T(x,y,z) = T_0\).

To calculate the overall heat transfer coefficient for surfaces in contact with water, \(U_w\), the combined effect of convective heat transfer coefficient of the circulating water, \(h_w\), and the thermal conductivity of the carton walls, \(k_c\), was taken into account. The correlation given by White (1988) for laminar flow past a flat plate in Eq. (6) and its useful variation Eq. (7) were used to calculate \(h_w\).

\[
Nu = \frac{h_w x}{k_w} = 0.332 Re^{1/2} Pr^{1/3}
\]  

\[
h_w = 0.332 \rho^{1/2} V_w^{1/2} k_w^{2/3} \frac{C_{pw}}{Pr} x^{-1/2} \mu_w^{-1/6}
\]  

where, \(Nu\), \(Re\) and \(Pr\) are the Nusselt, Reynolds, and Prandtl numbers, respectively, \(x\) is the characteristic length of the carton wall \((=0.095 \text{ m})\), and \(k_w\), \(C_{pw}\) and \(\mu_w\) are, respectively, the thermal conductivity, specific heat and viscosity of water calculated at the mean temperature of the water bath \((20\degree C)\). \(V_w\) is the velocity of water in the bath \((= 0.01 \text{ m/s})\). Equation (7) gave \(h_w = 123.7 \text{ W/m}^2\cdot\text{K}\). The overall heat transfer coefficient, \(U_w\), was calculated by taking the reciprocal of the sum of the convective and conductive thermal resistances.

\[
U_w = \frac{1}{1/h_w + \Delta x_c/k_c}
\]  

where, \(\Delta x_c\) is the thickness of the carton walls \((= 0.6 \times 10^{-3} \text{ m})\) and thermal conductivity of the carton wall, \(k_c = 0.13 \text{ W/m.K}\). Equation (8) gave \(U_w = 80.0 \text{ W/m}^2\cdot\text{K}\).

The overall heat transfer coefficient for the bottom surface was zero as it was insulated, while the overall heat transfer coefficient for the top surface \(U_a\) was equal to the heat transfer coefficient for air \(h_a\). \(h_a\) was calculated using the
correlation for natural convection over a flat plate given by White (1988).

\[ Nu = \frac{h_a x}{k_a} = 0.54 Ra^{1/4} \quad (9) \]

where \( k_a \) is the thermal conductivity of air and \( Ra \) is the Rayleigh number (product of the Grashoff number and Prandtl number). Equation (9) gave \( U_a = h_a = 1.49 \text{ W/m}^2\text{.K} \).

Biot number (\( Bi \)) was calculated in order to determine whether the temperature distribution would be significant.

\[ Bi = \frac{h_w x_w}{k} = \frac{h_w (b/4)}{k} = \frac{80 \left( \frac{0.095}{4} \right)}{0.575} = 3.30 \]

Since \( Bi > 0.1 \), spatial variation of temperature within the carton was expected. Therefore ANSYS finite element software was used to predict the temperature distribution in the fluid whole milk.

**Integration of ANSYS Transient Heat Transfer Analysis and BDGM**

Heat transfer model Eq. (4) and (5) were solved using the finite element method over a domain (consisting of the cube-shaped carton minus the headspace) discretized into 216 nodes and 125 elements. A trilinear eight-node hexahedral element was used to discretize the milk carton (Fig. 1). The discretized domain and boundary conditions are shown in Fig. 2.

Preliminary studies with different time step values, using the Crank-Nicolson time-marching scheme (Gerald and Wheatley 1999), showed that for a time step of more than 10 min the finite element solution did not converge. Therefore, a time step value of 5 min was chosen to be safer.

The space-time-temperature output obtained from ANSYS was input to the BDGM in an EXCEL™ spreadsheet format. The BDGM predicted microbial counts for each element of the discretized domain, and, from those counts, the average microbial counts at various times were calculated.

**Experiments for Validation of the IDG-FEM**

For validating the IDG-FEM, an experiment involving incubation of inoculated whole milk cartons in programmable water baths (221M, Neslab Inc.) was conducted. Raw milk and empty milk cartons were obtained from the
University Creamery, the Pennsylvania State University. Raw milk was sterilized (394.15K (121°C), 15 min) and then poured into half-gallon milk cartons that were sterilized by wiping with 70% ethanol.

Milk in the cartons was inoculated with *L. monocytogenes* to yield an initial level of approximately 3000 cfu/mL. The cartons were then sealed and placed in the water baths. At regular time intervals, samples (1 mL) were drawn from specific locations inside the milk cartons (1890 mL) and plated in duplicate on TSAYE (tripticase soy agar with 0.6% yeast extract). A separate milk carton was used at each sampling time. For average counts, individual containers were removed from the water baths and the milk was poured into a flask and mixed thoroughly on a stir plate before plating in duplicate on TSAYE.

The water baths were programmed to have time-varying temperatures, which corresponded to conditions existing in an hypothetical temperature-extreme storage facility (Fig. 3). Storage temperature fluctuated between 280.15 and 288.15K (7 and 15°C), with average value of 285.45K (12.3°C). Twice a day the temperature climbed to 301.15K (28°C) in a very short interval of time. To
represent times when the storage facility was exposed to outdoor conditions because of loading or unloading operations. The same pattern was repeated every 24 h for a total of 96 h or 4 days.

The convective boundary condition (which included the water bath time-temperature profile and the heat transfer coefficients) and initial microbial count in the milk carton were used as inputs for the IDG-FEM (Fig. 2). The IDG-
FEM validation consisted of subjecting cube-shaped cartons of milk (width (a) = depth (b) = 9.5 cm, height (c) = 18 cm, carton volume = 1.89 L) to the time-varying temperature condition shown in Fig. 3. Three replicate trials were conducted. A data acquisition system comprising the Campbell™ 21X Micro-logger and a PC was used to collect real-time data. Schematics of the validation experimental setup are given in Fig. 4 and 5. In-situ milk temperatures during

![Graph showing circulator bath temperature](image)

**FIG. 3. CIRCULATOR BATH TEMPERATURE FOR EXPERIMENTAL VALIDATION OF IDG-FEM**
FIG. 4. SCHEMATIC OF EXPERIMENTAL SETUP FOR VALIDATION OF IDG-FEM

The experiment were determined using thermocouple probes placed at various points (Fig. 6) in one uninoculated carton (placed in one of the water baths). Ten probes per container were used to measure the fluid milk's time-temperature cooling response. The storage time period was 96 h because the bacterial population reached its maximum stable value (due to increased competition for nutrients, etc.) within this time, before declining. In this study, only the growth phase of *L. monocytogenes* was modeled. The results of the simulation using the IDG-FEM were compared with those determined by the experiment.
RESULTS AND DISCUSSION

Comparison of Experimental and Predicted Temperatures in the Milk Carton

The results from ANSYS showed little spatial variation of predicted temperature within the milk carton, therefore, the time-temperature profiles of only six selected elements were used as input for the BDGM. Thermocouple probes 1 to 6 were used to measure temperatures inside the milk carton at the top-center (T Ce), geometric center (G Ce), bottom-center (B Ce), top-corner (T Co), mid-height corner (M Co) and bottom corner (B Co) positions shown in Fig. 6. Additional thermocouples monitored the water (probes 7 and 8) and air (probes 9 and 10) temperatures. The ANSYS program predicted the tempera-
turers at locations TCe, GCe, BCe, TCo, MCo and BCo with the following boundary conditions:

- Side surfaces: \( U_w = 80 \text{ W/m}^2\cdot\text{K} \); \( T_{\text{bulk},w} = \frac{(T_7 + T_9)}{2} \)
- Top surface: \( U_a = 1.59 \text{ W/m}^2\cdot\text{K} \); \( T_{\text{bulk},a} = \frac{(T_9 + T_{10})}{2} \)
- Bottom surface: insulated

where

\[ U_w = \text{overall heat transfer coefficient for surfaces in contact with water}, \]
\[ U_a = \text{overall heat transfer coefficient for surfaces in contact with air}, \]
\[ T_{\text{bulk},w} \text{ and } T_{\text{bulk},a} = \text{average temperatures of water and air}, \]
\[ T_7, T_8, T_9 \text{ and } T_{10} = \text{temperatures measured by probes 7, 8, 9 and 10 (Fig. 6)}. \]

The predicted and experimental temperatures at locations: TCe, GCe, BCe, TCo, MCo and BCo compared well with standard error ranging from 0.7 to 1.6K, while the maximum relative differences (MRDs) varied between 1.20 to 3.27%. The maximum relative difference and standard error were 1.77% and 1.62K, respectively. Comparisons for other five locations being similar to Fig. 7, they are not included.

The maximum absolute differences between predicted and measured temperature values at the six locations at any given time were less than 2.1K. Thus, the milk temperature within the carton had very little variation. Therefore, for predicting the average microbial counts inside the milk carton, the predicted temperatures at only the above six locations were used as input to the BDGM. Those locations represent extreme points within the milk carton. It is interesting to note that finite element analysis was used to model the transient heat transfer within the carton for Biot number \( >> 0.1 \), and, therefore, high internal resistance to heat transfer and significant spatial variation in temperature were expected. On the contrary, the experimental data and predicted values showed small spatial temperature variation within the milk carton. This can be explained by the fact that the difference between the mean temperatures of the carton and the surrounding water was always small (implying negligible internal resistance, \( Bi < 0.1 \), which means that maximum temperature difference observed within an object is never more than 10% of the difference between the initial temperature of the object and the temperature of the surrounding medium (White 1988)). This may not be the case for a different temperature profile in the water bath, and significant spatial variation in temperature might be observed.
Comparison of Experimental and Predicted Microbial Counts in the Milk Carton

Microbial counts were monitored at three locations — TCe, BCe and TCo (Fig. 6). Sampling was done at intervals ranging from 12 to 24 h. The microbial
growth data at locations Tce, Bce and Tco were very similar. This was expected because of the very little temperature variation within the carton as discussed in the previous section. Therefore, only average microbial growth data of the stated three points are reported in this paper.

FIG. 7. COMPARISON OF EXPERIMENTAL AND PREDICTED TEMPERATURES FOR LOCATION GCe (GEOMETRIC CENTER)
The predicted and experimental average microbial counts are given in Fig. 8. All experimental data points except for one were within the growth band predicted for $\alpha_0$ (initial physiological state parameter) between 0.05 and 0.99 for all locations. The mean temperature corresponding to the ambient temperature profile (Fig. 3) was 285.45K (12.3°C), and the corresponding $\alpha_0$ value was 0.132. In Fig. 8, the predicted growth curve for $\alpha_0$ of 0.132 is also shown. The experimental values of microbial counts for each location and the average count

![Graph showing growth curves with experimental data and predicted growth bands.](image)

**FIG. 8. EXPERIMENTAL AND IDG-FEM PREDICTED AVERAGE GROWTH OF L. MONOCYTOGENES IN A MILK CARTON**
were in general greater than the predicted values with $\alpha_0 = 0.132$. The standard error between the experimental and predicted values varied between 0.45 log (cfu/mL) to 0.62 log (cfu/mL), and the maximum relative difference varied between 14.27 to 16.28%.

Only one data point was slightly above the predicted maximum population density value of 7.47 log (cfu/mL), but this difference was not regarded as important because of the standard deviation of the experimental data (ranging between 0.21 to 0.35 log (cfu/mL), Alavi et al. 1999). Underprediction of microbial counts in dynamic temperature conditions by the IDG-FEM can be attributed to: (1) inability of the BDGM to represent accurately the lag phase of the inoculated cells (Alavi et al. 1999), and (2) possible migration of bacteria within the milk carton, contrary to the assumption that cells will remain stationary. The first factor may be eliminated by making a few modifications to the BDGM that should be explored (Alavi et al. 1999). The second factor will not be significant for solid and semisolid foods, but will remain a confounding factor for liquid foods. In either case, the IDG-FEM may still be used to provide a conservative estimate of growth of *L. monocytogenes* in milk cartons if a high value of $\alpha_0$ (closer to 1.0) is assumed.

**CONCLUSIONS**

Experiments were conducted to determine growth characteristics of *Listeria monocytogenes* in sterilized whole milk at nine temperatures in the range of 277.15 to 308.15K (4 to 35°C). Based on these data, the parameter values of the Baranyi dynamic growth model were statistically determined. Finite element software, ANSYS, was used to determine temperature distributions in milk cartons subject to a time-varying ambient temperature profile. The space-time-temperature data were input to the Baranyi dynamic growth model, to predict the microbial population density distribution and the average population density in the milk carton. Based on the comparisons of the IDG-FEM simulations and experimental data, the following conclusions were drawn from this study:

1. The IDG-FEM adequately predicted the spatial distribution of temperature and growth of *L. monocytogenes* in sterilized whole milk in a half-gallon laminated cardboard carton for time-varying ambient temperature. The standard error of the predicted temperatures ranged from 0.73 to 1.62K, while the maximum relative difference between the predicted and experimental temperatures ranged from 1.19 to 2.67%.

2. Experimental microbial counts predicted by the IDG-FEM were within the predicted band of growth. The growth curve at the mean $\alpha_0$ value under-
predicted microbial growth (standard error = 0.54 log (cfu/mL) and maximum relative difference = 15.49%).

(3) This study demonstrated the feasibility of using a FEM software in conjunction with a microbial growth model (BDGM) to predict microbial growth in a food product that had not been done in previous studies.

REFERENCES


